Crossing probabilities in Voronoi Percolation

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12 Oct 2015

A Voronoi tiling of the plane is obtained by first generating a point process η in the plane and then assigning to each point $x \in \eta$ the cell C_x consisting of those y which are closer to x than to any other point of η. Colouring cells red and blue independently with probability $1/2$ we obtain a Voronoi percolation configuration in the plane.

A straightforward symmetry argument shows that the probability of a red horizontal crossing of a square is precisely $1/2$. This symmetry argument breaks if one has already generated the Voronoi tiling. Could it be that the crossing event is very much dependent on the choice fo the point process η?

We prove the conjecture of Benjamini, Kalai and Schramm, which states informally that the randomness comes from the colouring rather than the choice of the point process. Specifically, writing H_n for the event of a horizontal red crossing of $[0, n]^2$, we prove that

$$\Pr(H_n|\eta) \xrightarrow{p} \frac{1}{2}\,.$$

We also discuss Noise Sensitivity properties of percolation, including the quenched Voronoi percolation model discussed above.

[Based on joint work with Daniel Ahlberg, Robert Morris and Vincent Tassion]