TOPOLOGICAL LOWER BOUNDS FOR COMPUTATION TREES AND ARITHMETIC NETWORKS

NICOLAI VOROBJOV

Abstract

We prove that the height of any algebraic computation tree for deciding membership in a semialgebraic set $\Sigma \subset \mathbb{R}^n$ is bounded from below by

$$\frac{c_1 \log(\mathrm{b}_m(\Sigma))}{m+1} - c_2 n,$$

where $\mathrm{b}_m(\Sigma)$ is the m-th Betti number of Σ with respect to "ordinary" (singular) homology, and $c_1,\ c_2$ are some (absolute) positive constants. This result complements the well known lower bound by Yao for *locally closed* semialgebraic sets in terms of the total *Borel-Moore* Betti number.

We also prove that if $\rho:\ \mathbb{R}^n \to \mathbb{R}^{n-r}$ is the projection map, then the height of any tree deciding membership in Σ is bounded from below by

$$\frac{c_1 \log(\mathrm{b}_m(\rho(\Sigma)))}{(m+1)^2} - \frac{c_2 n}{m+1}$$

for some positive constants $c_1,\ c_2$.

We illustrate these general results by examples of lower complexity bounds for some specific computational problems.

An analogous theory is developed for *arithmetic networks*, a computational model aimed to capture the idea of a parallel computation in its simplest form. Here we generalize lower bounds of Montaña, Morais and Pardo (who considered locally closed semialgebraic sets relative to Borel-Moore homology) to arbitrary semialgebraic sets relative to singular homology.

This is a joint work with Andrei Gabrielov (Purdue).